ISSN:2582-8169

"Study of K- Idempotent Fuzzy Matrix"

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Received:04th November, 2022; Revised:11thNovember,2022 Accepted :20th November 2022

Abstract: In matrix theory, an idempotent matrix is one that, when multiplied by itself, produces itself. The matrix must be square in order to determine the product. k-idempotent matrices are defined as a generalisation of idempotent matrices associated and inspired by the parameter k. When a fuzzy matrix A is obtained by k permuting the elements of A^2, it is said to be idempotent. Here k is the fixed product of disjoint transposition in S_n , the symmetric group of order n. The idempotent fuzzy matrix, also known as the idempotent F_k , matrix, is defined and characterised in this article. Some fundamental properties of idempotent fuzzy matrices are investigated. In this paper, we introduce and investigate the concept of k - Idempotent fuzzy matrix as a generalisation of Idempotent fuzzy matrix via permutations.

Keywords: **Idempotent fuzzy matrix, k – Idempotent fuzzy matrix, permutation matrix, zero patterns, commutator.**

I. INTRODUCTION

If and only if $A^2 = A$, a fuzzy matrix $A = [a_{ij}]_{n\times n}$is said to be idempotent. The concept of idempotent fuzzy matrices has been discussed by H. Y. Lee et al. [1].The parameter k - idempotent is associated and motivated by the notion of k - idempotent matrices

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Introduced by Krishnamoorthy et al.[3] as a generalisation of idempotent matrices; we introduce and study a new characteristic k - idempotent fuzzy matrix in this paper. A fuzzy matrix A is called k - idempotent if it is obtained by k - permuting the elements of A^2,. In this case, k is the fixed product of disjoint transposition in S_n, the symmetric group of order n. In this paper, some characterization of a k – idempotent fuzzy matrices are examined such as sum and product of two k-idempotent fuzzy matrices are k- idempotent. Furthermore, we show that some properties for k – idempotent fuzzy matrices which will be intended to provide further discussions. For a matrix $A = [a_{ij}]_{n\times n}$, $A^T, adj\,A$, and detA denotes the transpose, adjoint and determinant of the fuzzy matrix A. Let 'k' be a fixed product of disjoint transpositions in S_n , the set of all permutation on {1, 2,…, n}. Hence it is involuntary (that is k_2 = identity permutation). A square matrix is called a permutation matrix [4] if every row and every column contains exactly one '1' and all the other entries are '0' .In this paper, the index set {1,2,… n −1,n} will be denoted by N. By the Prop. 2.4.5 in [4],], $adjA=A^c$, where A^c is idempotent and $c \leq n-1$ The operations +, . and – are defined as follows:

$$a + b = \max\{a, b\}, \qquad a.b = \min\{a, b\}$$

$$a - b = \begin{cases} a if a > b \\ 0 \ if a \leq b \end{cases}$$

CHARACTERIZATIONS OF k – IDEMPOTENT FUZZY MATRICES

Definition 2.1 For a fixed product of disjoint transposition $k \in S_n$, a matrix $A = [a_{ij}]_{n\times n}$ is said to be k-idempotent if $KA^2 K = A$, where K is the associated permutation fuzzy matrix of 'k'. The associated permutation fuzzy matrix K is a matrix with one on its southwest – northeast diagonal and zeros everywhere else.

That is $K = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$.

Let $A = \begin{bmatrix} 10 & 1 & 1 \\ 4 & 10 & 2 \\ 4 & 6 & 8 \end{bmatrix}$

Remark 2.3

KA^2 K=A implies that KAK=A^2 . From the definition, the following relations can also be obtained which would be useful in computational aspect.

$K^2A = AK^2 = A$; $KA^3K = A^3$ and $KA^2K = A^2$

Theorem 2.4

A k – idempotent fuzzy matrix A is ciruculant [4] if and only if AK = KA.

Proof. Assume that AK = KA

Pre multiplying by K,

we have, KAK = A

But $A^2 = A$ {since A is k – idempotent}

Hence A is k – idempotent. The converse is also true by retracing the steps.

Next, we are examining some basic properties of idempotent fuzzy matrices. We know that all 1×1 fuzzy matrices are k – idempotent. Hence, in this paper, we deal only with square fuzzy matrix that dimension $n, n \geq 2$. Let F_1 be the set of all idempotent fuzzy matrices.

Theorem 2.5

A fuzzy matrix is k - idempotent if and only if all its zero patterns [1] are idempotent. By the above Lemma 2.4, we examine the properties of (0,1) fuzzy matrices and obtain a theorem and canonicalform of the

(0,1) fuzzy matrices. Thus we will be able to tocharacterize the structure of the set of all idempotent fuzzy matrices, F_1 .

Lemma 2.6

The set of all idempotent fuzzy matrices, F_1 is closed under the following operations

(i) Permutation similarity

(ii) Transposition.

Remark 2.7

The product of the permutation matrix K must be identity. i.e.,

$K^2 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} = I,$ the identity matrix

In particular if $k(i) = i$ then the associated permutation matrix K reduces to identity matrix and k-idempotent fuzzy matrix reduces to idempotent fuzzy matrix.

Lemma 2.8

Let$A = [a_{ij}]_{n\times n}$ be a k-idempotent fuzzy matrix. If $a_{ij} = 0$ for some i and j in N, then each product $a_{ik}a_{kj}$for all k in N.

Proof. It is an immediate consequence of the fuzzy matrix product.

Remark 2.9

Let A be k – idempotent fuzzy matrix, then A^T is also k – idempotent.

Proposition 2.11

If the fuzzy matrix A is k – idempotent, then adjA is also k – idempotent.

Proof. We know that C $AdjA$=A^c , where A^c is idempotent and $c \leq n$–1 [4]

Since A is k-idempotent, $KA^2K = A$

Also, $K(A^C)^2K = A^C$

Hence $A^C = AdjA$ is k-idempotent.

Proposition 2.12

If A is k – idempotent, then the fuzzy matrix $AadjA$ is also k – idempotent.

Proof

$$K(AadjA)^2K$$
$$K(AadjA)^2K$$
$$= K(A^2(adjA)^2)K$$
$$= K\,A^2K.K(adjA)^2K$$
$$= A\,AdjA$$

Hence $A\,AdjA$ is k-idempotent.

Lemma 2.13

Let A and B are two k – idempotent fuzzy matrices, then

$$det(A) + det(B) = \det(A + B)\; and \det(\mathrm{A}).\det(\mathrm{B}) = \det(A.B)$$

SOME OPERATIONS ON k-IDEMPOTENT FUZZY MATRICES

Proposition 3.1

Let A and B be two k-idempotent fuzzy matrices. Then A+B is k – idempotent fuzzy matrix. **Proposition 3.2**

Let A and B be two k-idempotent fuzzy matrices. If AB=BA, then AB is also a k-idempotent fuzzy matrix.

Proof.$K(AB)^2K$

$$= KA^2K.KB^2K$$
$$= A.B$$

Hence the fuzzy matrix AB is k-idempotent.

The following theorem gives the generalization of products of k – idempotent matrices.

Theorem3.3

If $A_1, A_2, A_3, \ldots, A_n$ be a k-idempotent fuzzy matrices belonging to a commuting family of matrices, then $\prod_{i=1}^{n} A_i$ is a k –

idempotent fuzzy matrix.

Proof. $K\left(\prod_{i=1}^{n} A_i\right)^2 K$

$= K(A_1.A_2.A_3 \;\; \dots.A_n)^2 K$

$= K({A_1}^2.{A_2}^2\dots{A_n}^2)K$

$= K{A_1}^2K.K{A_2}^2K \dots K{A_n}^2K$

$= A_1.A_2.A_3 \;\; \dots.A_n$

$= \prod_{i=1}^{n} A_i$

Hence the fuzzy matrix $\prod_{i=1}^{n} A_i$ is a k-idempotent.

Definition 3.4 For a pair of k – idempotent fuzzy matrices A and B, the commutator of A and B is denoted by [A,B] and defined by $[A,B] = AB - BA$.

Remark 3.5 If A and B are two k – idempotent fuzzy matrices then A+B is k – idempotent if and only if [A,B] = BA.

If A and B are two k – idempotent fuzzy matrices then AB is k – idempotent if and only if [A , B]= 0.

Theorem 3.7 If A and B are two k – idempotent fuzzy matrices then A(A+B)B commutes with the permutation matrix K.

Proof. $A(A+B)B$

$= A^2B + AB^2$

$= KA^2KB + AKB^2K$

$= KAB^2K + KBA^2K$

$= K(AB^2 + BA^2)K$

$= KA(A+B)BK$

Hence KA(A+B)B = A(A+B)BK.

Theorem 3.8 Let A and B are two commuting k – idempotent fuzzy matrices. The k – idempotency of A (A+B)B necessarily implies that is a null matrix.

Proof For any two k – idempotent fuzzy matrices A and B, we have A(A+B)B commutes with the permutation matrix K in proposition 3.2.

If A(A+B)B is k – idempotent then by Lemma 3.6, it reduces to an idempotent matrix. i.e., $[A(A+B)B]^2 = A(A+B)B$ -------- (3.1)

i.e., $[BA^2] + (AB^2)^2 + A^2BAB^2 + AB^2A^2B = A^2B + AB^2$

Since A and B are k – idempotent fuzzy matrices, we have $A^4 = A$ and $B^4 = B$.

Hence (3.1) becomes, $A^2B + AB^2 + A^3B^3 + A^3B^3 = A^2B + AB^2$

i.e., $2A^3B^3 = 0$

i.e., $A^3B^3 = 0$

i.e., $(AB)^3 = 0$-------- (3.2)

Since A and B are commuting k – idempotent fuzzy matrices, AB is also k – idempotent by Hence $(AB)^4 = AB$.

Pre – multiplying equation (3.2) by AB, $(AB)^4 = 0$ i.e., AB = 0 It follows that A (A+B) B=0.

CONCLUSION

Clearly, studying canonical forms of this type is essential for developing fuzzy matrix theory. The concept of a k-idempotent fuzzy matrix is extended to periodic matrices or n-potent fuzzy matrices, i.e.,$KA^nK = A$.

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